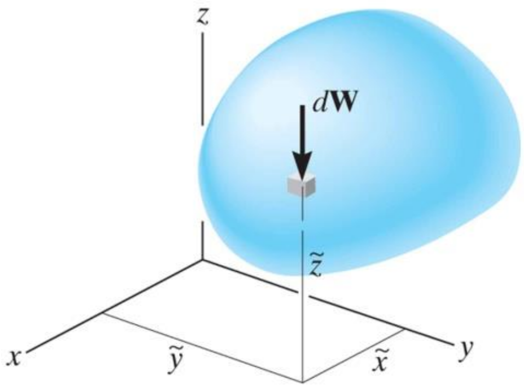
Chapter 9: Center of Gravity and Centroid

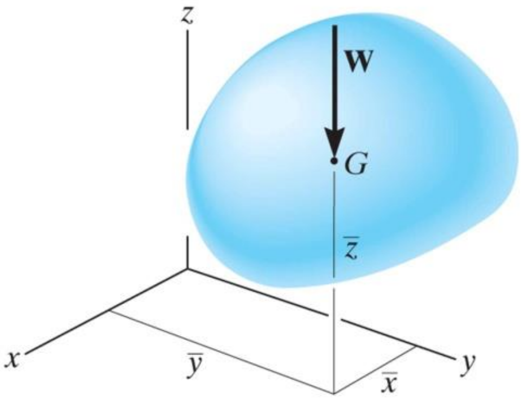
Center of gravity

To design the structure for supporting a water tank, we will need to know the weight of the tank and water as well as the locations where the resultant forces representing these distributed loads act.

How can we determine these resultant weights and their lines of action?

Center of gravity





A body is composed of an infinite number of particles, and so if the body is located within a gravitational field, then each of these particles will have a weight *dW*.

The **center of gravity (CG)** is a point, often shown as G, which locates the resultant weight of a system of particles or a solid body.

From the definition of a resultant force, the sum of moments due to individual particle weight about any point is the same as the moment due to the resultant weight located at G.

If *dW* is located at point $(\tilde{x}, \tilde{y}, \tilde{z})$ then

$$\bar{x}\,W = \int \tilde{x}\,\mathrm{dW}$$

$$\bar{y}\,W = \int \tilde{y}\,\mathrm{dW}$$

$$\bar{z}\,W = \int \tilde{z}\,\mathrm{dW}$$

Center of Mass

$$\bar{x} = \frac{\int \tilde{x}\, dm}{\int dm}$$

$$\bar{y} = \frac{\int \tilde{y}\, dm}{\int dm}$$

$$\bar{z} = \frac{\int \tilde{z}\, dm}{\int dm}$$

Center of Volume

$$\bar{x} = \frac{\int \tilde{x}\, dV}{\int dV}$$

$$\bar{y} = \frac{\int \tilde{y}\, dV}{\int dV}$$

$$\bar{z} = \frac{\int \tilde{z}\, dV}{\int dV}$$

Center of Area

$$\bar{x} = \frac{\int \tilde{x}\, dA}{\int dA}$$

$$\bar{y} = \frac{\int \tilde{y}\, dA}{\int dA}$$

$$\bar{z} = \frac{\int \tilde{z}\, dA}{\int dA}$$

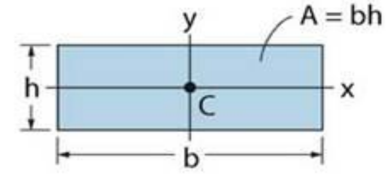
Centroid

The centroid, C, is a point defining the geometric center of an object.

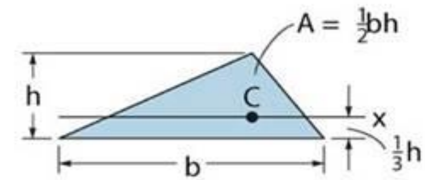
The centroid coincides with the center of mass or the center of gravity only if the material of the body is homogenous (density or specific weight is constant throughout the body).

If an object has an axis of symmetry, then the centroid of object lies on that axis.

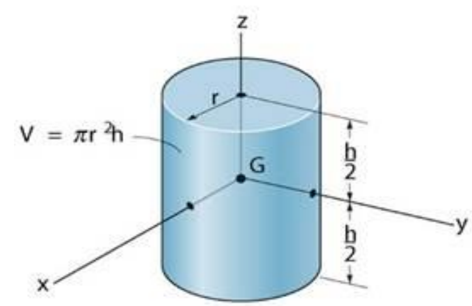
In some cases, the centroid may not be located on the object.



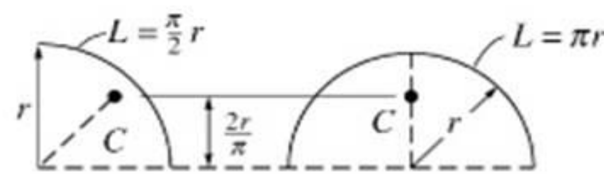
Rectangular area



Triangular area

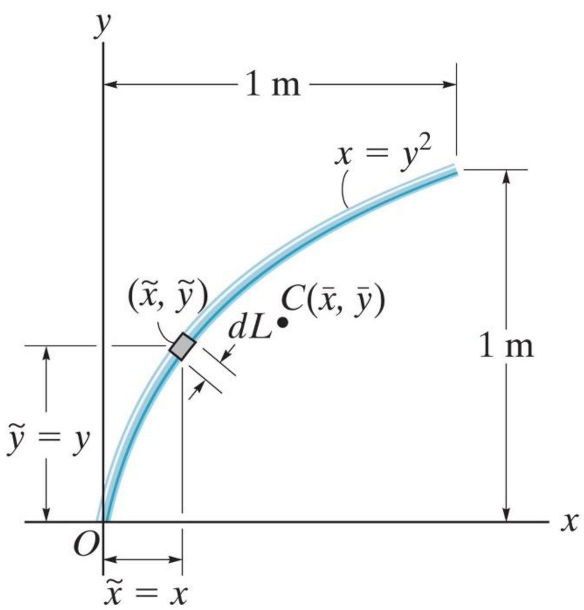


Cylinder

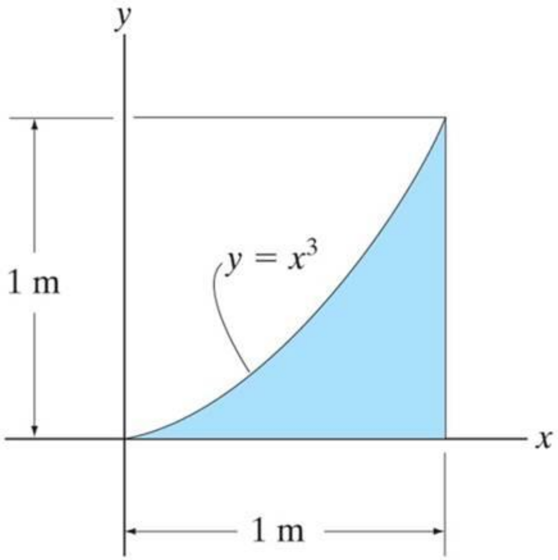


Quarter and semicircle arcs

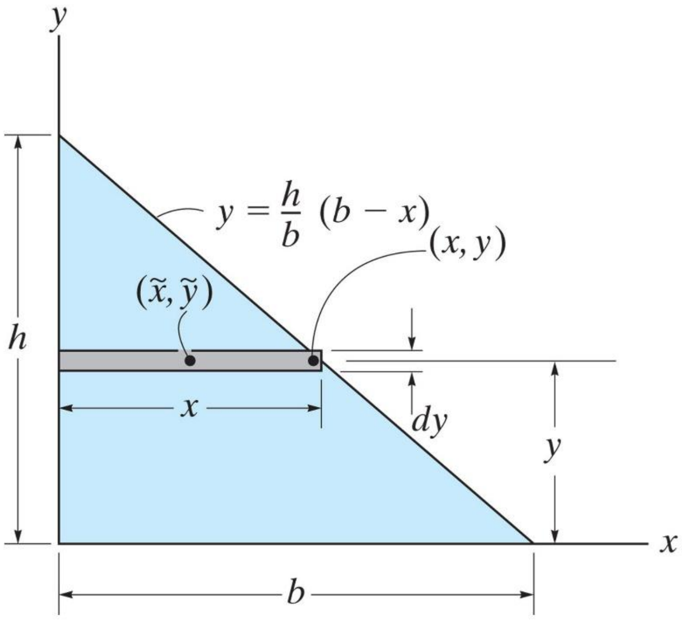
Find the centroid of the rod bent into the shape of a parabolic arc



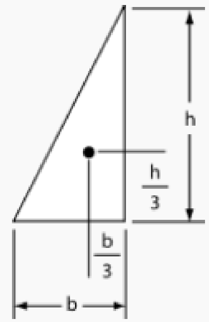
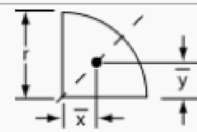
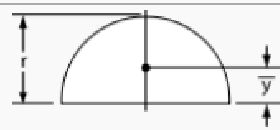
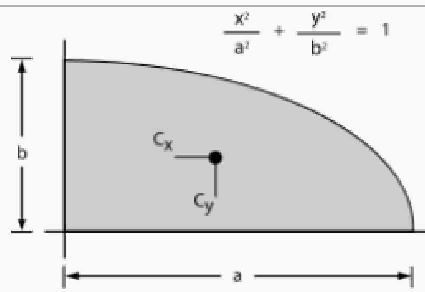
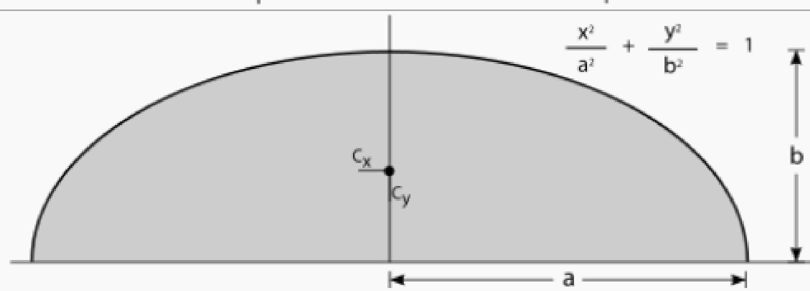
Find the centroid of the area below



Find the centroid of the area below

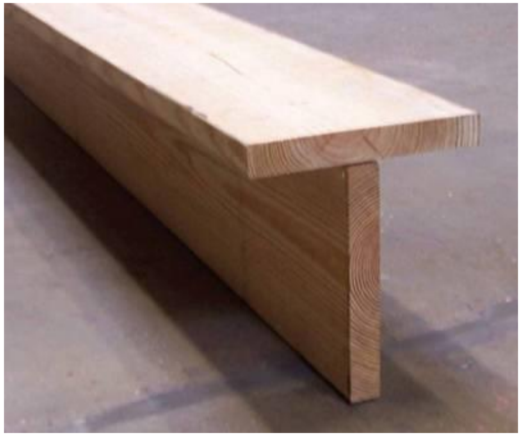


Centroid of typical 2D shapes

Shape	Figure	$\bar{x}$	$\bar{y}$	Area
Right-triangular area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$

http://en.wikipedia.org/wiki/List_of_centroids

Applications

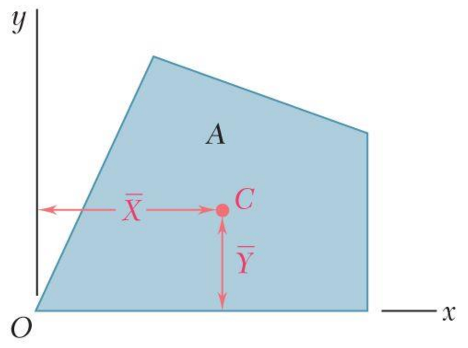


The I-beam (top) or T-beam (bottom) shown are commonly used in building various types of structures.

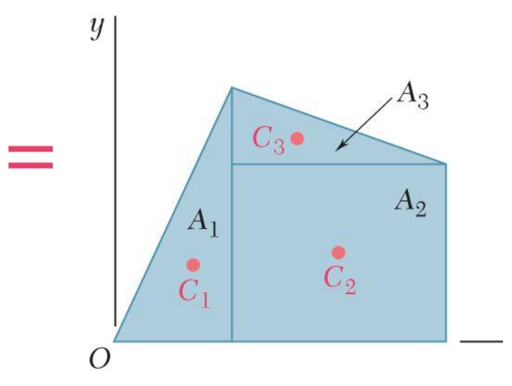
How can we easily determine the location of the centroid for different beam shapes?

Composite bodies

A composite body consists of a series of connected simpler shaped bodies. Such body can be sectioned or divided into its composite parts and, provided the weight and location of the center of gravity of each of these parts are known, we can then eliminate the need for integration to determine the center of gravity of the entire body.



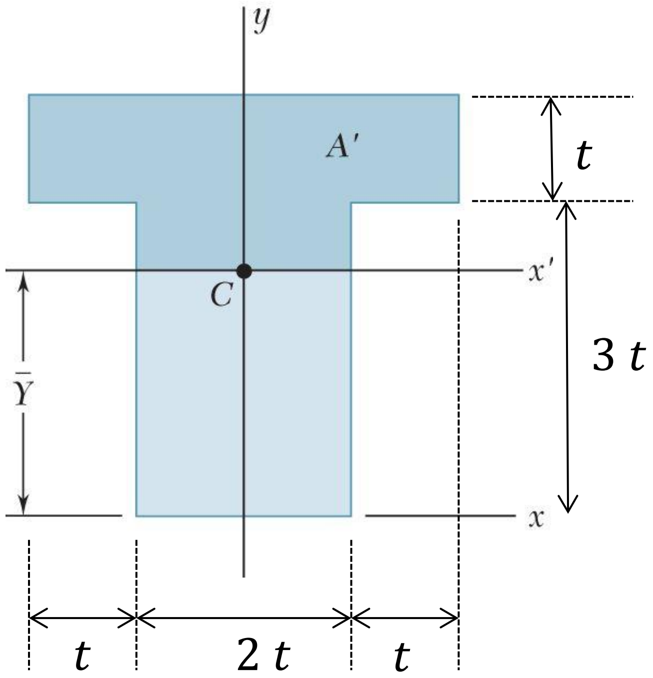
For example, the centroid of the area A is located at point C of coordinates $\bar{x}$ and $\bar{y}$. In the case of a composite area, we divide the area A into parts A_1, A_2, A_3



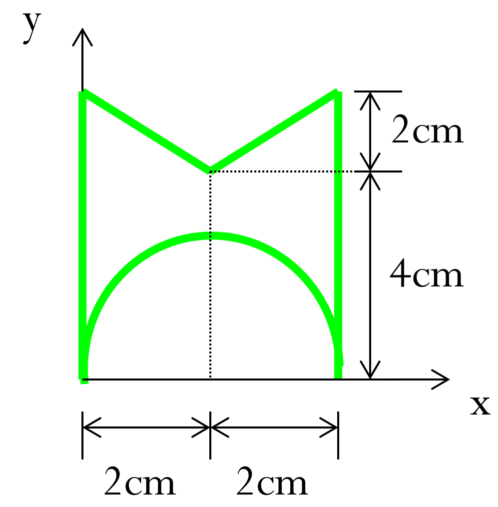
$$A_{total}\,\bar{X} = \sum_i A_i\,\bar{x}_i$$

$$A_{total}\,\bar{Y} = \sum_i A_i\,\bar{y}_i$$

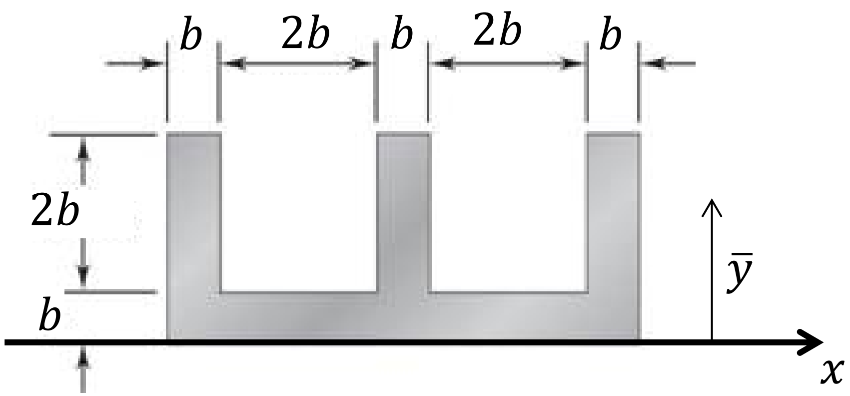
Find the centroid of the area below.



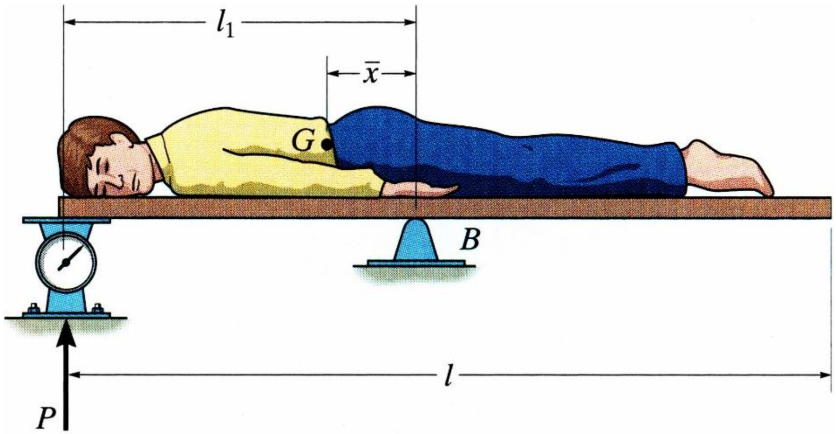
A rectangular area has semicircular and triangular cuts as shown. What is the centroid of the resultant area?

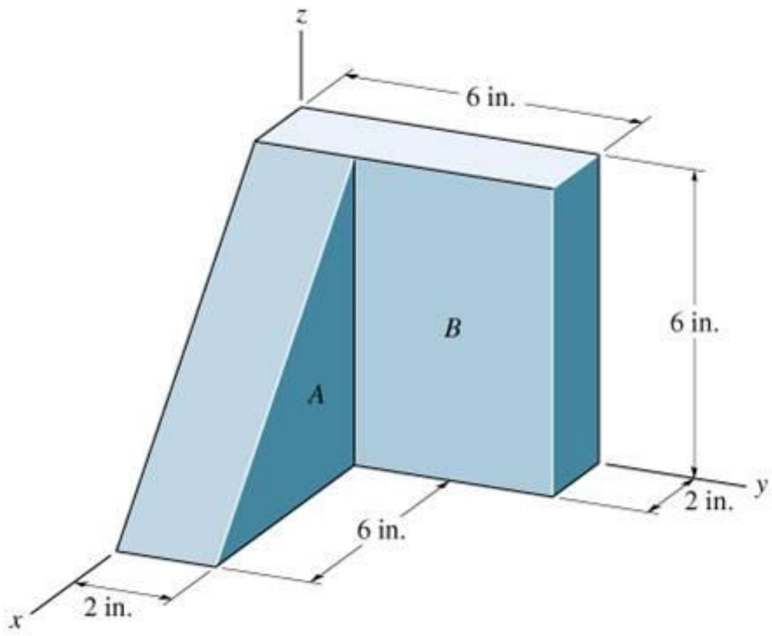


Find the centroid of the area below.



The anatomical center of gravity G of a person can be determined by using a scale and a rigid board having a uniform weight W_1 and length l. With the person's weight W known, the person lies down on the board and the scale reading P is recorded. From this, show how to calculate the location of the centroid $\bar{x}$. Discuss the best place l_1 for the smooth support at B in order to improve accuracy of this experiment.





Two blocks of different materials are assembled as shown. The densities of the materials are:

$$\rho_A = 150 \text{ lb / ft}^3 \quad \text{and}$$
$$\rho_B = 400 \text{ lb / ft}^3.$$

Find: The center of gravity of this assembly.

Determine the location of the center of gravity of the three-wheeler. The location of the center of gravity of each component and its weight are tabulated in the figure. If the three-wheeler is symmetrical with respect to the x-y plane, determine the normal reaction each of its wheels exerts on the ground.

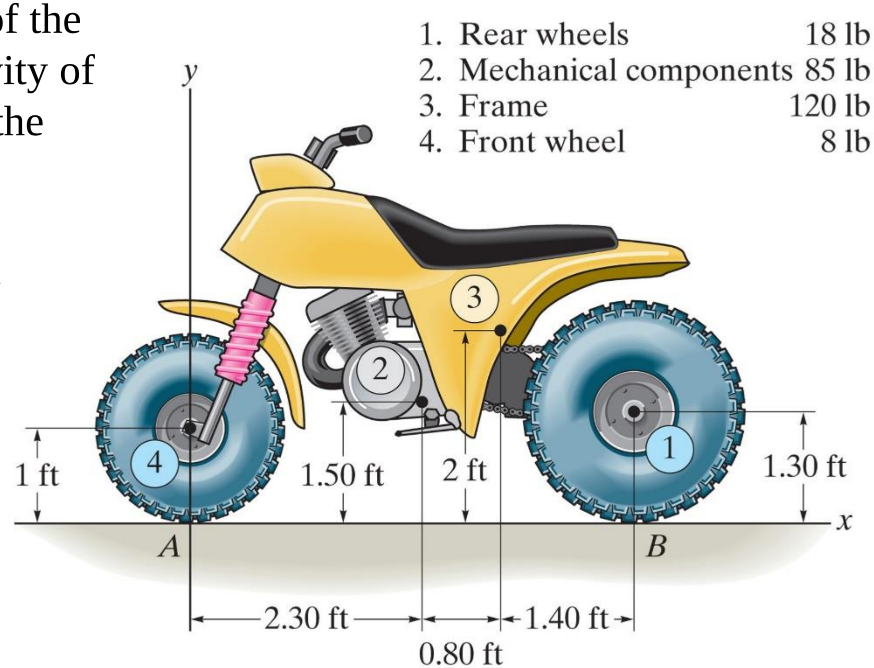


Figure: 09_P077